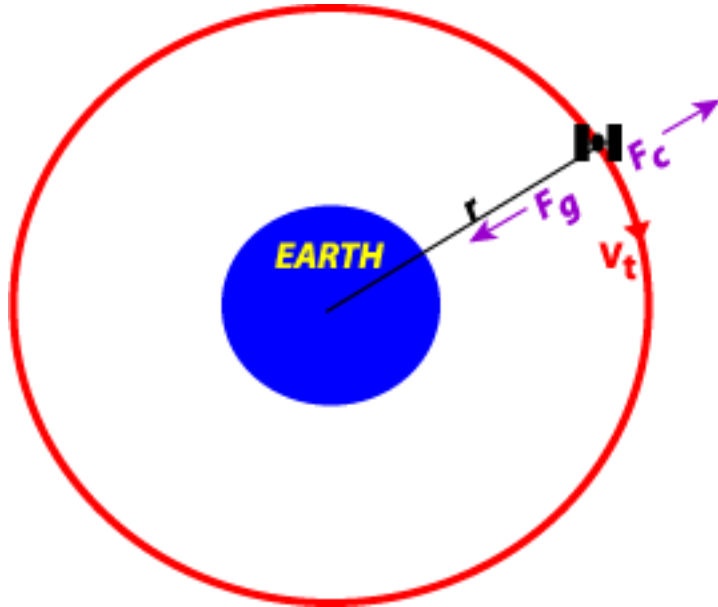


## THEORY FOR GOES ORBIT

Q: Why are GOES 35,800 km from the center of the earth so that they are in geosynchronous orbit???

There are two relevant forces involved in this problem:



1. **gravitational force** of attraction between any two objects, given by:
2. **centrifugal force** an outward-directed force that normally balances the inward-directed centripetal force,

given by: . These forces are required to help maintain the circular trajectory of an object.

In our situation of a satellite in geosynchronous orbit, the outward-directed centrifugal force balances the inward-directed gravitational force. Hence, for a steady-state orbit, the force balance becomes:

$$\text{or} \quad (1)$$

Solving for  $v_s$ , the tangential velocity of the satellite, from (1) yields:

$$(2)$$

Notice, that in (2), the mass of the satellite does not appear.

**REALITY CHECK** – what are we trying to solve for?????

OK, so what is  $v_s$ ?

The tangential velocity of the satellite ( $v_s$ ) is related to its orbital period,  $T$  through:

$$\text{or} \quad \text{or} \quad (3)$$

Eliminating  $\omega_s$  between (3) and (2) gives:

Solving for the orbital period,  $T$ , gives:

$$(4)$$

OK, we still do not know  $r$ .....but we're getting closer. To find  $r$ , we still need to determine what  $T$  is.....

What is the constraint, in terms of angular velocity, on the satellite if it is to be in a geosynchronous orbit?????

Yes, where  $\omega_s$  and  $\omega_e$  are the angular velocities of the satellite and earth, respectively.

The angular velocity (from basic physics) for the satellite is:

$$(5)$$

but from (3), recall that  $\omega_s = \frac{v}{r}$  or

$$(6)$$

Substituting (6) into (5) gives:

$$\text{or solving for } T, \quad \omega_s = \omega_e$$

$$(7)$$

recall that  $\omega_e = \frac{2\pi}{T_e}$  so (7) can be rewritten as:

$$(8)$$

From (8), we now know the satellites orbital period,  $T$ .

By substituting (8) into (4) to eliminate  $T^2$  we get:

$$\text{or solving for } r \text{ yields:}$$

$$(9)$$

We know:

$$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$$

$$m_e = 5.97 \times 10^{24} \text{ kg}$$

$$\omega_e = 7.29 \times 10^{-5} \text{ rad s}^{-1}$$

Hence, substituting the above constants into (9) gives:

$$R = 35,786 \text{ km for GOES}$$

There it is.....

